Introduction

Despite long research history and multiple successful applications the forward modelling of elastic waves propagating in anisotropic heterogeneous 3D media still exceeds time and cost limitations of routine use in the industry. Anisotropy is observed in real data because of thin layered, fracturing and intrinsic structure. Complex tectonics, salt domes and reefs, fractured carbonates in combination with anisotropy create unfavourable conditions for application of physically simplified approaches and 2D methods. Numerical wavefield simulation for each shot deals with cubes of size hundreds wavelengths. From implementation viewpoint it is important to subdivide the problem by smaller independent parts to minimize communication and synchronization overheads. Time domain approaches require about 10 measures per wavelength to preserve wavelet. To propagate 9 wavefield components (3 partial displacement velocities or shifts, 3 compression stresses and 3 shear stresses) one needs hundreds gigabytes per shot. Frequency domain approaches can benefit from use of sparse models. However, in most cases numeric simulation is applied for analysis of small inhomogeneities and thin layered zones improper to high frequency ray tracing approximation. This results in similar memory requirements. Forward modelling review of Verieux et al. (2009) shows that 3D time domain simulation mostly surpasses frequency domain methods despite their ability to reuse inversed impedance matrix for multiple shots. The state-of-the-art 3D elastic modelling was discussed by Petersson (2009), Lisitsa and Vishnevskiy (2010). Lisitsa and Vishnevskiy proposed a memory efficient finite-difference solution of 3D elastic anisotropic problem.

Diverse efforts were put in acceleration of 3D elastic anisotropic modelling by use of specialized hardware/software tools. Komatitsch et al. (2010) reported 25-50 times acceleration achieved on NVIDIA GPU cluster under CUDA. Lavreniuk et al. (2011) discussed grid technology application. Nevertheless the computational cost of the problem considerably exceeds practical limitations.

3D modelling can be simplified if the medium properties are fixed along some direction. Such type models are called 2.5D ones. One source line computed in 2.5D model can be replicated to produce full 3D survey. Thus the total problem size is decreased dozen times. Besides, model itself is described by 2D arrays instead of 3D ones. Transition from 3D to 2.5D model makes it impossible to estimate illumination of target horizons or optimize acquisition geometry. However, such tasks are pretty good solved by ray tracing. 2.5D forward modelling instead helps to investigate benefits from multi-component survey, effects of inaccuracy in anisotropy estimation, resolution of a processing batch etc. Variety of 2.5D modelling applications were described in Kostyukevych and Roganov (2010). Here we discuss computational benefits of the 2.5D approach, synergy of 2.5D with GPU and applicability of interpolation techniques for additional acceleration.

Method

Song and Williamson (1995) reduced the problem of 2.5D acoustic modelling for a constant density medium to a linear system of Fourier-transform equations for time and Y variables. They solved the system of equations by LU-decomposition of the right part matrix. Cao and Greenhalgh (1998) deduced stability conditions and proposed the unilateral equation for suppression of reflection at absorbing boundaries. Neto and Costa (2006) have proposed a 2.5D simulation method for elastic isotropic and anisotropic media. However, they have applied the theory only for isotropic and transversally isotropic medium. 2.5D implementations for arbitrary 3D TTI anisotropy were presented by Silva Neto, Costa and Novais (2007) and Kostyukevych et al. (2008).

3D elastic anisotropic system of equations expresses Hooke's law combined with Newton's 2nd law:

$$\begin{cases} \rho(X)\dfrac{\partial u_n(X,t)}{\partial t} = \dfrac{\partial \tau_{nk}(X,t)}{\partial x_k} + f_n(X,t) \\ \dfrac{\partial \tau_{mn}(X,t)}{\partial t} = \Lambda_{mnkq}(X)\dfrac{\partial u_k(X,t)}{\partial x_q} + \dfrac{\partial M_{mn}(X,t)}{\partial t} \end{cases} \tag{1}$$

where $X = (x_1 \quad x_2 \quad x_3)$ means a point in 3D space, u_n denote displacement velocities, τ_{nm} are components of a stress tensor. Parameters of the geology media are stiffness tensor Λ and density ρ. Source signal is encoded by the vector functions of source forces f_n and moment forces M_{nm}. Both type forces are zero out of source.

In 2.5D case for arbitrary y $\Lambda(x,y,z) = \Lambda(x,0,z)$ and $\rho(x,y,z) = \rho(x,0,z)$. So it is convenient to represent the wavefield in Fourier domain by its decomposition along the axis x_2:

$$u_n(X,t) = \int_{-\infty}^{+\infty} \widetilde{u}_n(\widetilde{X},t) e^{i\omega y} dy \, , \; \tau_{mn}(X,t) = \int_{-\infty}^{+\infty} \widetilde{\tau}_{mn}(\widetilde{X},t) e^{i\omega y} dy \tag{2}$$

where $\widetilde{X} = (x_1 \quad \omega \quad x_3)$. The system (1) can be re-written in the space Fourier domain ($q \in \{1,3\}$):

$$\begin{cases} \rho(\widetilde{X}) \dfrac{\partial \widetilde{u}_n(\widetilde{X},t)}{\partial t} = \dfrac{\partial \widetilde{\tau}_{nq}(\widetilde{X},t)}{\partial x_q} + i\omega \widetilde{\tau}_{n2}(\widetilde{X},t) + \widetilde{f}_n(\widetilde{X},t) \\ \dfrac{\partial \widetilde{\tau}_{mn}(\widetilde{X},t)}{\partial t} = \Lambda_{mnkq}(X) \dfrac{\partial \widetilde{\tau}_k(\widetilde{X},t)}{\partial x_q} + \Lambda_{mnk2}(X) i\omega \widetilde{u}_2(\widetilde{X},t) + \dfrac{\partial \widetilde{M}_{mn}(\widetilde{X},t)}{\partial t} \end{cases} \tag{3}$$

Numeric wave propagation simulation according to (3) in the form of the second order central finite-difference scheme on three staggered grids was described in Kostyukevych et al. (2008).

To avoid numerical dispersion and aliasing one has to use reasonably big space frequency diapason and dense sampling. Lavreniuk et al. (2011) have shown the computational complexity cut down because of frequency decomposition (3) is moderate:

$$Speedup \approx W \Bigg/ \left(\frac{1}{2} + \frac{V_{\max} t_{\max}}{y_{\max} - y_{\min}} \right) \tag{4}$$

where $W = 5..15$ is the minimal number of grid points per wavelength. Time of recording $t_{\max}$, max velocity $V_{\max}$, diapason of crossline offsets from $y_{\min}$ and $y_{\max}$ are problem dependent. For deep target objects $t_{\max}$ is big and under fixed offsets theoretical speedup (4) becomes smaller than 1. In other words computational complexity can be higher than for full 3D solution of the same problem. But the memory requirements are decreased dramatically. 2.5D uses just 160 bytes per cell of 2D grid to store both the wavefield state and the model for single space frequency. Quasy-2D sub-problems of different space frequencies are solved independently. As result 2.5D simulation task can be easy divided by hundreds or thousands parts which can be computed by GPU cards in the most efficient mode: without data reload, using single floating point precision and local calculations only. Space domain results are obtained by inverse Fourier transform of the quasy-2D solutions. The inverse Fourier transform is perfectly scalable and fits to GPU implementation by parts.

Additional acceleration of 2.5D method can be obtained for theoretical models which contain relatively small number of interfaces. In such cases seismograms are sparse in space domain including lines along the axis x_2. Hennenfent and Herrmann (2008) have shown that random/jittered undersampling of a space domain signal which sparse in frequency domain generates low amplitude noise in spectrum. The same statement is right back direction. In sparse 2.5D case one can compute just a random portion of quasy-2D seismograms. The noise can be removed then by linear threshold. Interpolation instead of simple ignoring the absent frequencies improves the method.

Experiments

Tests (Fig. 1) show expected preference of modern GPUs over outdated GTX 8800. Mid level GTX480 unexpectedly approached professional Tesla. Probably it's because of single precision math. Speedup has been estimated relative to single core of CPU Intel Xeon E5345 (2.33GHz).

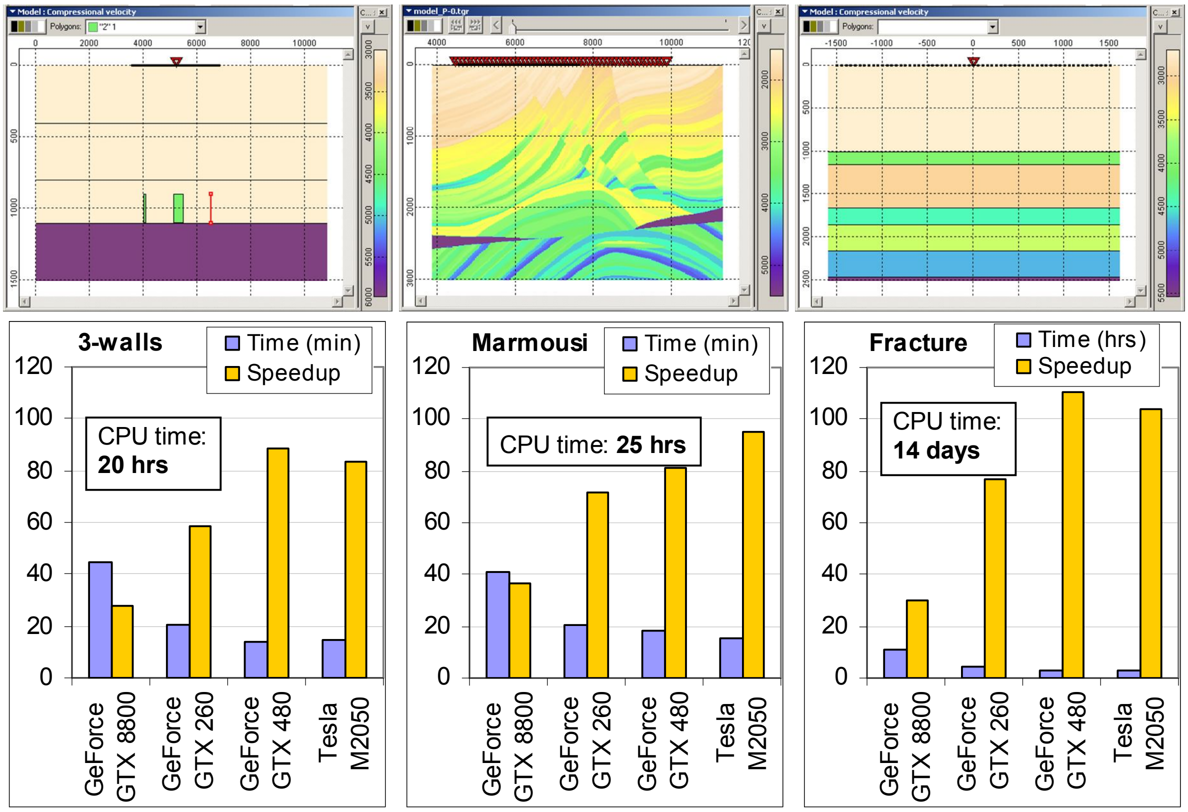

Figure 1: *GPU calculation time and speedup for models "3-walls", "Marmousi", "Fracture".*

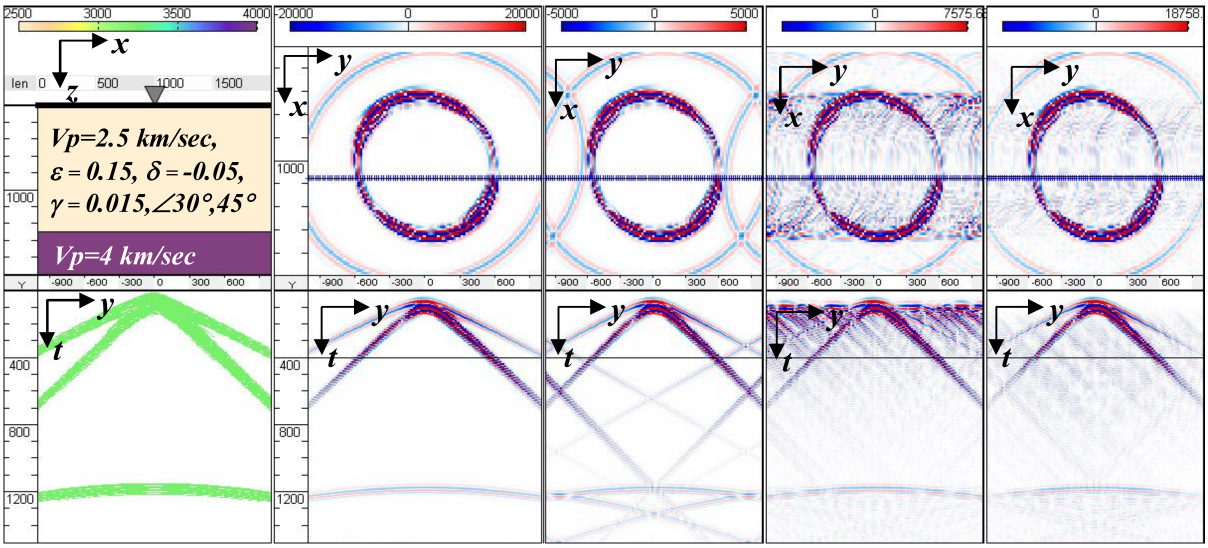

Figure 2: *Model and signal mask (left column), correct sampling, regular undersampling, random undersampling, random undersampling with spectrum interpolation (right column).*

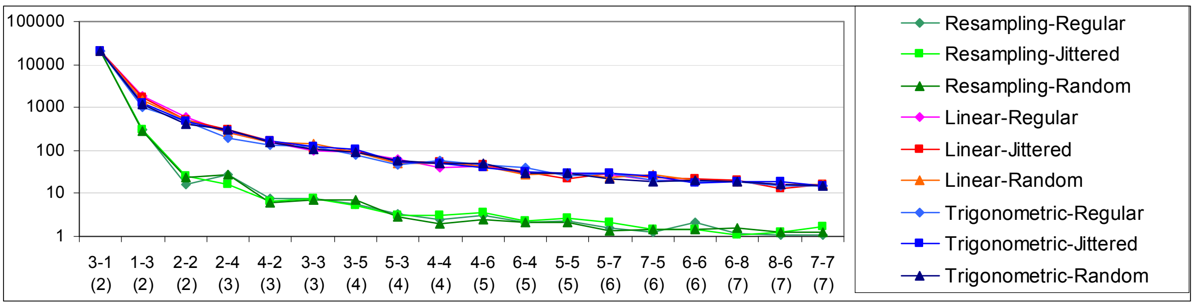

Figure 3: *Signal to noise ratio for 3 undersampling and 3 interpolation methods.*

Figure 2 illustrates the random undersampling on a simple 2.5D model with 3D TTI anisotropy. The synthetic seismograms were generated by simulation of 2 sec wavefield propagation in a cube of size 2.5 km in each direction. The simulation grid cell is 2.5 m. In full 3D case one need 216 GB to place both wavefield and model. In full 2.5D case 581 quasy-2D problems of size 432 MB has been solved. Simulation complexity is about 2 times smaller then for 3D (but acceleration is much higher). Results are shown in the 2nd column. Regular 4 times undersampling (3rd column) results in strong noise from two mirror sources. Random 4 times undersampling (4th column) dissipates the mirror signals. But noise amplitudes left relatively big, especially for small times. Random 4 times undersampling with trigonometric local interpolation of spectrum provides low noise. (Signal to noise energy ratio is 105).

Figure 3 demonstrates efficiency of random undersampling for various parameters. Interpolation of frequencies stable improve signal to noise energy ratio for about an order.

Conclusions

2.5D elastic anisotropic method of finite-difference forward modelling can generate realistic 3D synthetic seismograms for geometrically simplified but detailed models with arbitrary 3D TTI anisotropy and fracturing. Both high level and low level GPUs provide high performance for 2.5D.

Additional 3-4 times acceleration can be obtained for simple models by random undersampling with spectra interpolation for the cost of about 1% noise.

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