P042

Quasi-acoustic Approximations for qSV-waves in a Transversely Isotropic Medium

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SUMMARY

We derive "quasi-acoustic" approximations valid for qSV-wave propagation in transversely isotropic and tilted transversely isotropic media by applying approximation extracted from acoustic approximation for qP-wave propagation. One approximation has the same accuracy as qP-wave acoustic approximation for the same range of horizontal slowness, the other approximations are wide-angle approximations.

INTRODUCTION

There are many approximations to the slowness surface in a transversely isotropic (VTI) medium both for qP- and qSV-waves. The quasi-acoustic approximation introduced by Akhalifah (1998) is of the great help for processing, modeling and interpretation of qP-wave data in a VTI medium. The number of parameters to define the qP-wave propagation in quasi-acoustic approximation is significantly reduced comparing with exact expression, while the approximation is practically accurate for realistic values of anisotropy parameters.
In our paper we define the quasi-acoustic approximation not as approximation to the medium parameters but as the special type of approximation to the slowness surface. We start with analyzing of the slowness surface for qP-waves, extract the approximation, which results in quasi-acoustic approximation, and then extend this approach to the slowness surface for qSV-waves. By doing so, we obtain the series of "quasi-acoustic" approximations for qSV-waves. The accuracy of approximations is compared for a simple VTI model. We also illustrate the applicability of "quasi-acoustic" approximation for traveltime computation in a tilted transversely isotropic medium.

QUASI-ACOUSTIC APPROXIMATION FOR qP WAVES

The quasi-acoustic approximation was proposed by Alkhalifah (1998) for qP-wave seismic data processing. The basic idea behind this approximation is setting the vertical qSV-wave velocity to zero ($\beta_0 = 0$). Since β_0 is excluded, only three parameters are required to define qP-wave propagation in the VTI medium. These parameters are: the vertical qP-wave velocity α_0 and Thomsen' s (1986) anisotropy parameters ε and δ. For imaging of qP-wave data, these parameters are composed into the normal moveout velocity $v_{PP} = \alpha_0\sqrt{1+2\delta}$ and parameter $\eta = (\varepsilon-\delta)/(1+2\delta)$. In quasi-acoustic approximation the equation for vertical slowness for qP-wave takes the form

$$q_\alpha^2 = \frac{1}{\alpha_0^2}\left[1 - \frac{p^2 v_{PP}^2}{1-2\eta p^2 v_{PP}^2}\right], \tag{1}$$

where p is horizontal slowness.

THE qP-WAVE QUASI-ACOUSTIC APPROXIMATION

The vertical slowness squared for qP- and qSV-waves can be given as (Stovas and Ursin, 2003)

$$\mathrm{q}_\alpha^2 = \frac{1}{\alpha_0^2}\left[1-\left(1+2\delta+\mathrm{S}(\mathrm{p})\right)\mathrm{p}^2\alpha_0^2\right], \quad \mathrm{q}_\beta^2 = \frac{1}{\beta_0^2}\left[1-\left(1+2\sigma-\mathrm{S}(\mathrm{p})\right)\mathrm{p}^2\beta_0^2\right], \tag{2}$$

where $\sigma = (\varepsilon-\delta)/\gamma_0^2$, $\gamma_0^2 = \beta_0^2/\alpha_0^2$, and

$$\mathrm{S}(\mathrm{p}) = \frac{4}{\left(1-\gamma_0^2\right)} \frac{\sigma\left(1-\gamma_0^2+2\delta\right)\mathrm{p}^2\beta_0^2}{1+\frac{2}{1-\gamma_0^2}(\delta-\sigma)\mathrm{p}^2\beta_0^2+\sqrt{\mathrm{Q}(\mathrm{p})}},$$
$$\mathrm{Q}(\mathrm{p}) = 1+\frac{4}{1-\gamma_0^2}(\delta-\sigma)\mathrm{p}^2\beta_0^2+\frac{4}{\left(1-\gamma_0^2\right)^2}\left[2\left(1-\gamma_0^2\right)\sigma+(\delta+\sigma)^2\right]\mathrm{p}^4\beta_0^4. \tag{3}$$

The approximate behaviour of $\mathrm{S}(\mathrm{p})$ at $\mathrm{p}=0$ is $\mathrm{S}(\mathrm{p}) \approx 2\sigma\gamma_0^2(1+2\delta)\mathrm{p}^2\alpha_0^2$. Let us approximate function $\mathrm{S}(\mathrm{p})$ for the range of horizontal slowness $\left[0, \mathrm{P}_{\mathrm{qP}}\right]$ corresponding to qP-wave propagating region such that to preserve both the behavior at $\mathrm{p}=0$ and limit of $\mathrm{S}(\mathrm{p})$ at $\mathrm{p}=\mathrm{P}_{\mathrm{qP}}$, $\mathrm{S}\left(\mathrm{p}=\mathrm{P}_{\mathrm{qP}}\right) = 2\sigma\gamma_0^2$.

$$\hat{S}(p) = \frac{2\sigma\gamma_0^2(1+2\delta)p^2\alpha_0^2}{1-2\sigma\gamma_0^2 p^2\alpha_0^2}. \tag{4}$$

Substituting approximation (4) into equation (2) results exactly in the same expression derived by Alkhalifah (1998) for acoustic approximation (see equation (1)).

QUASI-ACOUSTIC APPROXIMATIONS OF qSV-WAVE

If the classic acoustic approximation is defined not by setting β_0 to be zero, but by the approximation of $S(p)$ given in equation (4), we can derive completely new equations for qSV-wave propagation. To be used for qSV-wave, the approximation (4) has to be rewritten as

$$\hat{S}_1(p) = \frac{2\sigma(1+2\delta)p^2\beta_0^2}{1-2\sigma p^2\beta_0^2}, \tag{5}$$

Let us keep approximate behaviour of function $S(p)$ at $p=0$ and preserve value of $S(p)$ at $p = P_{qSV}$ (corresponding to horizontal propagation of qSV waves). It gives the new approximation for function $S(p)$

$$\tilde{S}_2(p) = \frac{2\sigma(1+2\delta)p^2\beta_0^2}{1+2\delta p^2\beta_0^2}. \tag{6}$$

The third "quasi-acoustic" qSV-wave approximation can be obtained by analyzing the phase slowness surface in a VTI medium (including both qP and qSV surfaces) which can be written as

$$\left((1+2\varepsilon)p^2 + q^2 - \frac{1}{\alpha_0^2}\right)\left(p^2+q^2-\frac{1}{\beta_0^2}\right) + 2(\varepsilon-\delta)p^2\alpha_0^2 q^2\left(\frac{1}{\beta_0^2}-\frac{1}{\alpha_0^2}\right) = 0. \tag{7}$$

If point in the slowness space (p,q) belongs to qP-wave slowness surface, the following approximation can be used: $p^2 + q^2 \approx 1/\alpha_0^2$. Substituting this approximation into equation (7) and reducing the multiplier $\left(1/\beta_0^2 - 1/\alpha_0^2\right)$ results in the classical qP-wave quasi-acoustic approximation (1). Similar, if point belongs to qSV-wave slowness surface, the following expression is approximately valid, $(1+2\varepsilon)p^2 + q^2 \approx 1/\beta_0^2$. This approximation being substituted into equation (7) also results in reducing the multiplier $\left(1/\beta_0^2 - 1/\alpha_0^2\right)$, and finally gives $q_\beta^2 \approx \left(1/\beta_0^2\right)\left(1-p^2\beta_0^2\right)/\left(1+2\sigma p^2\beta_0^2\right)$, which can be considered as a new "quasi-acoustic" approximation for qSV waves. Similar to quasi-acoustic approximation (1), this approximation has two parameters only. From this approximation we can compute the corresponding function $S(p)$ as follows

$$\hat{S}_3(p) = \frac{2\sigma(1+2\sigma)p^2\beta_0^2}{1+2\sigma p^2\beta_0^2} \tag{8}$$

TRAVELTIME-OFFSET AND PHASE VELOCITY EQUATIONS

The offset and traveltime equations for qSV-wave can be given using the aneliptic function $S(p)$ as follows (Ursin and Stovas, 2006)

$$x(p) = p\beta_0^2 t_0 \frac{1+2\sigma - S(p) - pS'(p)/2}{\sqrt{1-p^2\beta_0^2\left(1+2\sigma - S(p)\right)}}, \quad t(p) = t_0 \frac{1-p^3\beta_0^2 S'(p)/2}{\sqrt{1-p^2\beta_0^2\left(1+2\sigma-S(p)\right)}}, \tag{9}$$

where t_0 is two-way vertical traveltime for qSV-wave and $S'(p) = dS(p)/dp$. Substituting corresponding functions $S(p)$ from the quasi-acoustic qSV-wave approximations results in different expressions for offset and traveltime parametric equations.
The qSV-wave phase velocity and phase angle equations can be computed by

$$v_S^2(p) = \frac{\beta_0^2}{1 - p^2\beta_0^2\left(2\sigma - S(p)\right)}, \quad \sin^2\psi(p) = \frac{p^2\beta_0^2}{1 - p^2\beta_0^2\left(2\sigma - S(p)\right)}. \qquad (10)$$

Using the approximations (5, 6 and 8), results in the different phase velocity approximations. All phase velocity approximations have no anomalous behavior as it was shown in Grechka et al. (2004) for qSV-wave in qP-wave acoustic approximation. Note that in weak-anisotropy approximation, all phase velocity approximations converge to the weak-anisotropy approximation from exact phase velocity squared equation (Thomsen, 1986).

qSV-WAVE SLOWNESS SURFACE FOR TTI MEDIUM

Exact and approximate equations $F(p_1, p_2, p_3) = 0$ which define the slowness surface for TTI medium with the symmetry axis $\mathbf{u}_3 = (\cos\varphi\sin\alpha,\ \sin\varphi\sin\alpha,\ \cos\alpha)^T$ can be obtained from equation (2) with use of exact (3) and approximate (5, 6 and 8) values of $S(p)$ by the variables change $p = \sqrt{w_1^2 + w_2^2}$ and $q = w_3$, where

$$\begin{aligned} w_1 &= p_1\cos\varphi\cos\alpha + p_2\sin\varphi\cos\alpha - p_3\sin\alpha \\ w_2 &= -p_1\sin\varphi + p_2\cos\varphi \\ w_3 &= p_1\cos\varphi\sin\alpha + p_2\sin\varphi\sin\alpha + p_3\cos\alpha \end{aligned} . \qquad (11)$$

To find explicit expression for $p_3(p_1, p_2)$ we have to solve equation $F(p_1, p_2, p_3) = 0$ for p_3. It can be done by Newton/Raphson-like iterative procedure. The initial value is chosen for isotropic medium with shear velocity β_0. This method can be implemented in computation of first arrivals in heterogeneous TTI medium using the finite-difference method (for example, upwind ENO Runge-Kutta method), as it has been shown in Roganov (2006) for acoustic qP-waves.

NUMERICAL RESULTS

To test the qSV-wave "quasi-acoustic approximations given in equations (5, 6 and 8, namely approximations 1, 2 and 3, respectively) we use single layer VTI model with parameters: $\alpha_0 = 2.0\,\mathrm{km/s}$, $\beta_0 = 1.0\,\mathrm{km/s}$, $\varepsilon = 0.1$ and $\delta = 0.05$. In Figure 1 we show the errors in vertical slowness, traveltime and phase velocity for qSV-wave computed from quasi-acoustic approximations. One can see that the approximation 1 performs the best in the range of horizontal slowness which correspond to the qP-wave propagation range. For larger values of horizontal slowness, the error in approximation 1 rapidly increases. Approximation 3 performs better then approximation 2. It is seen that the approximation 1 is extremely accurate for traveltime up to offset/depth equal 1. For larger offset it is very difficult to choose the best approximation. For small phase angles approximation 1 performs the best. For the entire range of phase angles, one can choose approximation 3.
The accuracy of quasi-acoustic approximation for qSV-wave we illustrate on the example with TTI medium with parameters mentioned above and the symmetry axis being located in the plane X_1X_3 and tilted with $\alpha = 30^\circ$. The source is located at $x_1 = 0$, $x_2 = 0$ and $x_3 = 1000\mathrm{m}$. In Figure 2 one can see the error in first arrival traveltime ($\tau_{exact} - \tau_{appr}$) computed with the finite-difference scheme using exact and approximate (8) equations for vertical slowness. One can see that errors in traveltime are relatively small.

CONCLUSIONS

We derive the series of “quasi-acoustic” approximation for qSV-waves, which do not require setting the vertical qP-or qSV-wave velocity to be zero, with the special type of approximation to the slowness surface. The approximations are compared for vertical slowness, traveltime and phase velocities in one numerical example. We show that approximations have different accuracy depending on the range of horizontal slowness (offset and phase angle) and number of the approximation parameters. The approximation 1 is very accurate for the range of horizontal slowness corresponding to qP-wave propagation. Therefore, it can be used for processing and modeling of qSV-waves within the C-wave experiment. The approximations 2 and 3 are valid for the entire range of horizontal slowness for qSV-wave, while approximation 3 has less number of parameters. We illustrate the applicability of “quasi-acoustic” approximation for traveltime computation in TTI medium.

ACKNOWLEDGEMENTS

Alexey Stovas would like to acknowledge the ROSE project for financial support.
Yuriy Roganov would like to acknowledge Tetra Seis Inc for financial support.

REFERENCES

Alkhalifah T. [1998] Acoustic approximations for processing in transversely isotropic media. *Geophysics* **63**, 623-631.

Grechka V., Zhang L. and Rector W.J. [2004] Shear waves in acoustic anisotropic media. *Geophysics* **69**, 576-582.

Roganov Y. [2006] 3D eikonal solver in tilted TI media. *68th EAGE Conference and Exhibition*, Extended Abstract.

Stovas A., and Ursin B. [2003] Reflection and transmission responses of layered transversely isotropic viscoelastic media. *Geophysical Prospecting* **51**, 447-477.

Thomsen L. [1986] Weak elastic anisotropy. *Geophysics* **51**, 1954-1966.

Ursin B., and Stovas A. [2006] Traveltime approximations for a layered transversely isotropic medium. *Geophysics* **71**, D23-D33.

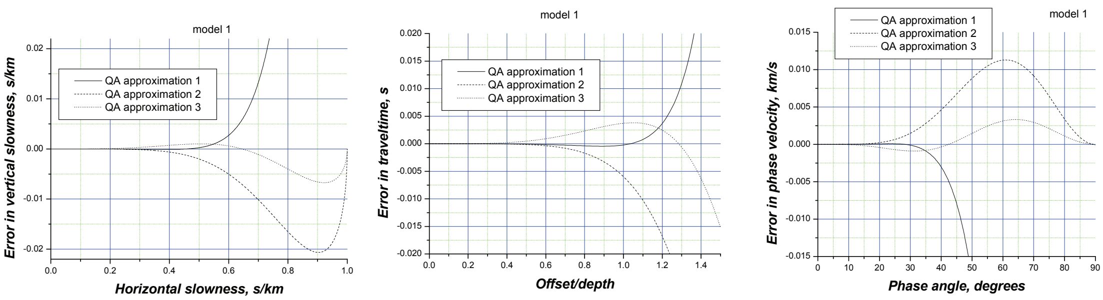

Figure 1. Error in vertical slowness (to the left), traveltime (in the middle) and phase velocity (to the right) from quasi-acoustic qSV-wave approximations (VTI model).

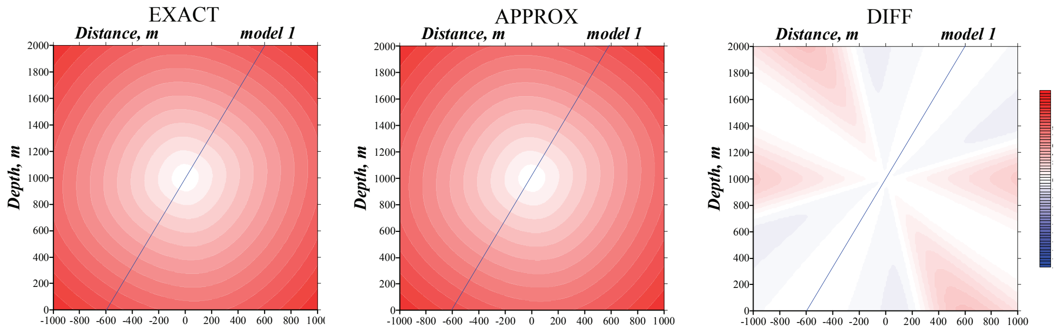

Figure 2. Exact (to the left), approximate (in the middle) traveltime and traveltime errors (to the right) from quasi-acoustic qSV-wave approximation 3 (TTI model).